conditional sentences and causal reasoning

seminar 2: Linking Pearl and Lewis

Katrin Schulz
ILLC
University of Amsterdam
COURSE PLAN

Conditional sentences and causal reasoning

GENERAL INTRODUCTION

Lecture 1: The logic of conditionals: The standard view

Tutorial 1: Challenges for the similarity approach

Lecture 2: Bayes Nets and Causal Bayes Nets

Tutorial 2: Counterfactuals as Interventions

SPECIFIC TOPICS

Seminar 1 (Practicum, part 1): 3 challenges for the framework
(I will first introduce the challenges individually, then you can choose and work in groups on one of them for 40 minutes)

Seminar 2: The relation between the similarity approach and the causal approach

Seminar 3: Using Logic Programming to model causal inferences

Seminar 4 (Practicum, part 2): presentations and discussion
(each group will shortly present their ideas, we will discuss them and I will comment on the state of the arts on each of the challenges)
Conditionals and Causality

Plan this class

• How does Lewis’ order work?
• Does Lewis’ order give the same result as Pearl’s approach?
• Can Pearl’s approach be translated into an (antecedent-independent) similarity order?
• What is the logical relation between the similarity approach and the structural equation approach?
“The counterfactual

(1)If Nixon had pressed the button there would have been a nuclear holocaust.

is true or can be imagined to be so. Now suppose that there never will be a nuclear holocaust. Then that counterfactual is, on Lewis' analysis, very likely false. For given any world in which the antecedent and consequent are both true it will be easy to imagine a closer world in which the antecedent is true and the consequent false. For we need only imagine a change that prevents the holocaust but that does not require such a great divergence from reality."

–Fine 1975: 452
Lewis’ solution

1. It is of the first importance to avoid big, widespread, diverse violations of law.

2. It is of the second importance to maximise the spatiotemporal region throughout which perfect match of particular fact prevails.

3. It is of the third importance to avoid even small, localised, simple violations of law.

4. It is of little or no importance to secure approximate similarity of particular fact, even in matters that concern us greatly.
How does Lewis’ order work?
1. avoid big violations of law.
2. maximise the spatiotemporal region of perfect match
3. avoid small violations of law.
Lewis

Nixon pushes button

undo button

holocaust takes place

Pearl

U1 → B → H

U2 → C → H
Lewis

w4<w3<w5<w1, w4

Nixon pushes button

undo button

holocaust takes place

Pearl

U1

B

H

U2

C

H
Lewis

w4 < w3 < w5 < w1, w4

Nixon pushes button

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U1

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C
Lewis

w4 < w3 < w5 < w1, w4

Nixon pushes button

undo button

holocaust takes place

Pearl

U1

B

U2

C

H
Conditionals reason about a particular subclass of the models that make the antecedent true.

2 possible approaches

- by similarity
- by construction

Lewis
miracles

Pearl
intervention

???
Does Lewis’ order give the same result as Pearls approach?
There are three lights called S1, S2 and L. The lights regularly change from on to off and back again. After observing the lights for a while you realize that there is a certain pattern in the way they are lit: either all lamps are on or only one of the lamps is on. At the moment S1 is off, S2 is on and L is off.

(5) If S1 had been on, L would have been on. (S1 → L)
Suppose there is a circuit such that the light is on (L) exactly when both switches are in the same position (up or down). At the moment switch one is down (¬S1), switch two is up (S2) and the lamp is off (L).

(4) If switch one had been up, the lamp would have been on. (S1 > L)
Observation
Conditionals reason along causal dependencies.

Interpretation
- Sensibility to causal dependence is an epiphenomenon (Lewis)
- The truth conditions of conditionals depend on causal dependencies (Pearl)
conditionals \[\rightarrow\] causality

**Lewis**

conditionals $\rightarrow$ causality

BUT: counterexamples
BUT: How to get the information about the true conditionals?

**Pearl**

causality $\rightarrow$ conditionals

BUT: How to get causal information?
Lewis’ solution

Lewis solution (order) doesn’t work well for standard examples in the literature and the way they are modelled.

Why isn’t \( w_1 \) a better world than \( w_2 \)?
Can Pearl’s approach be translated into an (antecedent-independent) similarity order?
similarity & structural equations

Result (Marti & Pinosio)

Yes.

This order captures two principles:

1. past facts are more important than future facts
2. you rather give up the law than the value of the causes
a kind of premise semantics …

… but now the premises are ordered according to their relevance for the similarity order.

Let $P$ be the premise set and $P_{w_0}(w)$ the set of premises that $w$ and $w_0$ agree on (they assign the same truth value).

1. standard premise semantics:
   
   $w_1 \preceq_{w_0} w_2$ iff $P_{w_0}(w_2) \subseteq P_{w_0}(w_1)$

2. ordered premise semantics:
   
   $w_1 \preceq_{w_0} w_2$ iff $P^*_{w_0}(w_2) \subseteq P^*_{w_0}(w_1)$, where $P^*_{w_0}$ is the highest ranked premise on which $w_1$ and $w_2$ differ.
but which premises and how are they ordered?

\[ Y_1 = X_1 \land X_2 \]
\[ Z = \neg Y_1 \land Y_2 \]
What is the logical relation between the similarity approach and the structural equation approach?
similarity & structural equations

Question 1:

What do we learn from the result of Marti & Pinosio about the logical relation between the similarity approach and structural models?
The logic of recursive structural models is as least as strong as that of the similarity approach, but only for the restricted language that recursive structural models are defined for.
The Logic of the similarity approach
What about this one?

(SDA): \(((\phi \lor \psi) \sim \chi) \rightarrow (\phi \sim \chi) \land (\psi \sim \chi)\)

- Is this principle valid in the system P? (Think about it from a semantic point of view.)
- What do you think about the intuitive plausibility of the principle?
The logic of recursive structural models is as least as strong as that of the similarity approach, but only for the restricted language that recursive structural models are defined for.

atomic sentences: $X=x$

conditionals: $(X_1=x_1 \land X_2=x_2 \land \ldots \land X_n=x_n) \rightarrow \phi$
Question 2:
Can we single out the class of similarity models that have the same logic as the class of recursive structural models?
Recursive similarity models have the same validities as do recursive structural models, but only for the restricted language that recursive structural models are defined for.
Recursive similarity models have the same validities as do recursive structural models, but only for the restricted language that recursive structural models are defined for.
Result 2

Recursive similarity models have the same validities as do recursive structural models, but only for the restricted language that recursive structural models are defined for.
Result 2

Recursive similarity models have the same validities as do recursive structural models, but only for the restricted language that recursive structural models are defined for.

- A similarity model is **recursive** iff for each world $w$ there is a total ordering $<_w$ on the variables such that if $X<_w Y$, then, keeping the value of all other variables constant, the closest world $v'$ where $Y=y$ is true assigns the same value to $X$ as the closest world where $Y$ has a different value.
similarity & structural equations

Question 3:
Can we generalise the result of Marti & Pinosio to arbitrary structural models?
similarity & structural equations

Result 3

No.

There is a formula that does not involve disjunction and is valid in the class of similarity models, but not in the class of causal models where the equations have a unique solution.
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There is a formula that does not involve disjunction and is valid in the class of similarity models, but not in the class of causal models where the equations have a unique solution.

$\neg \phi$ with $\phi = (A \sim (B \land \neg C)) \land (B \sim (C \land \neg A)) \land (C \sim (A \land \neg B))$
Proof (sketch)

1. We show that the sentence is not valid in the class of structural modals with a unique solution, by showing that …

\[ \neg \phi \text{ with } \phi = (A \rightarrow (B \land \neg C)) \land (B \rightarrow (C \land \neg A)) \land (C \rightarrow (A \land \neg B)) \]
Proof (sketch)

1. We show that the sentence is not valid in the class of structural modals with a unique solution, by showing that ... there is a structural model $M$ with $M \models \phi$.

$\neg \phi$ with $\phi = (A \rightarrow (B \land \neg C)) \land (B \rightarrow (C \land \neg A)) \land (C \rightarrow (A \land \neg B))$
Proof (sketch)

1. We show that the sentence is not valid in the class of structural modals with a unique solution, by showing that … there is a structural model \( M \) with \( M \vDash \phi \).

2. We show that the sentence is valid in the class of similarity models by showing that …

\[ \neg \phi \text{ with } \phi = (A \sim (B \land \neg C)) \land (B \sim (C \land \neg A)) \land (C \sim (A \land \neg B)) \]
Proof (sketch)

1. We show that the sentence is not valid in the class of structural modals with a unique solution, by showing that … there is a structural model $M$ with $M \models \phi$.

2. We show that the sentence is valid in the class of similarity models by showing that … there is no similarity model $M$ with $M \models \phi$.

\[ \neg \phi \text{ with } \phi = (A \mapsto (B \land \neg C)) \land (B \mapsto (C \land \neg A)) \land (C \mapsto (A \land \neg B)) \]
Reflection

The sentence is valid within the similarity framework, because the logic of the similarity approach validated LOOP. (P. Santorio)

A↝B
B↝C
C↝A

A↝C
basic idea

“If I scratched this match, it would light.”
(Goodman)

(1) If Jones had won the election, Smith would have retired to private life.
(2) If Smith had died last year, Jones had won the election.
(3) If Smith had died last year, Smith would have retired to private life.
basic idea

“If I scratched this match, it would light.”
(Goodman)

(1) \(((φ \rightarrow ψ) \land (ψ \rightarrow χ)) \rightarrow (φ \rightarrow χ))\), i.e. transitivity is not valid.
Question 3:
Can we generalise the result of Marti & Pinosio to arbitrary structural models?

Result 3
No.
There is a formula that does not involve disjunction and is valid in the class of similarity models, but not in the class of causal models where the equations have a unique solution.
Love triangle.
Andy, Billy, and Charlie are in a love triangle. Billy is pursuing Andy; Charlie is pursuing Billy; and Andy is pursuing Charlie. Each of them is very annoyed by their suitor and wants to avoid them. There’s a party going on and all three were invited. None of them ended up going, but each of them kept track of whether the person they liked was going. Each of them wanted an occasion to spend time with their beloved and without their suitor. Having an occasion of this kind would have been sufficient for each of them to go.

(22) If Andy was at the party, Billy would be at the party.  
(23) If Billy was at the party, Andy would not be at the party.  

—Paolo Santorio
References

primary texts:

- Lewis, D.: 1979, `Counterfactual dependence and time's arrow'. NOUS 13, 455-476.